Chapter 13. Filter Inductor Design

13.1. Several types of magnetic devices, their *B*-*H* loops, and core vs. copper loss

13.2. Filter inductor design constraints

13.3. The core geometrical constant K_g

13.4. A step-by-step design procedure

13.5. Summary of key points

13.1. Several types of magnetic devices, their *B-H* loops, and core vs. copper loss

A key design decision: the choice of maximum operating flux density B_{max}

- Choose B_{max} to avoid saturation of core, or
- Further reduce B_{max}, to reduce core losses

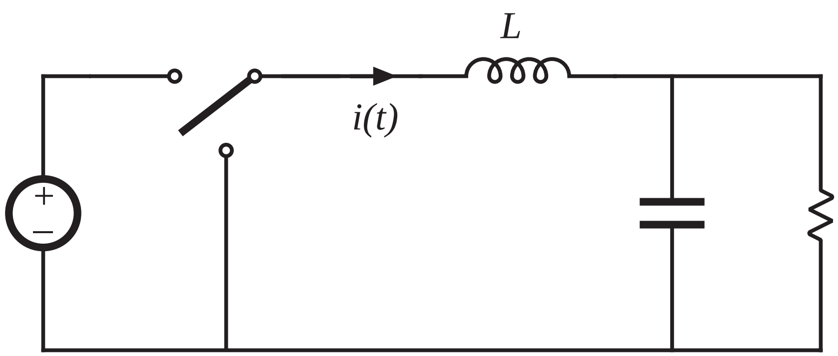
Different design procedures are employed in the two cases.

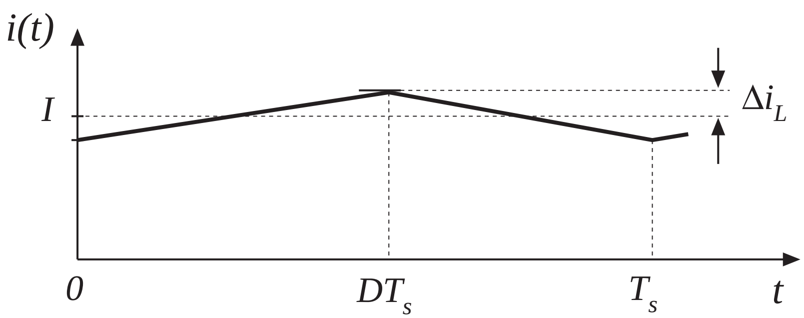
Types of magnetic devices:

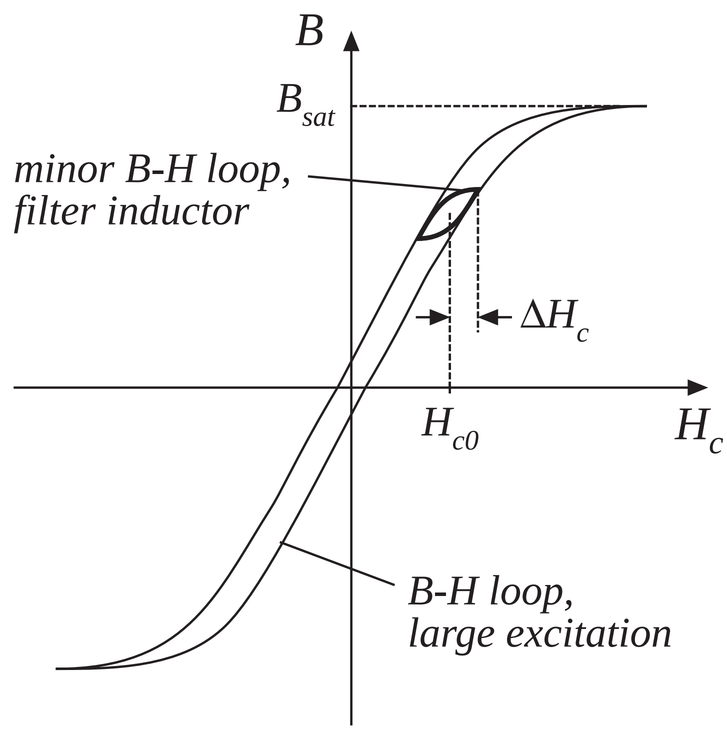
Filter inductor	Ac inductor
Conventional transformer	Coupled inductor
Flyback transformer	SEPIC transformer
Magnetic amplifier	Saturable reactor

Filter inductor

CCM buck example

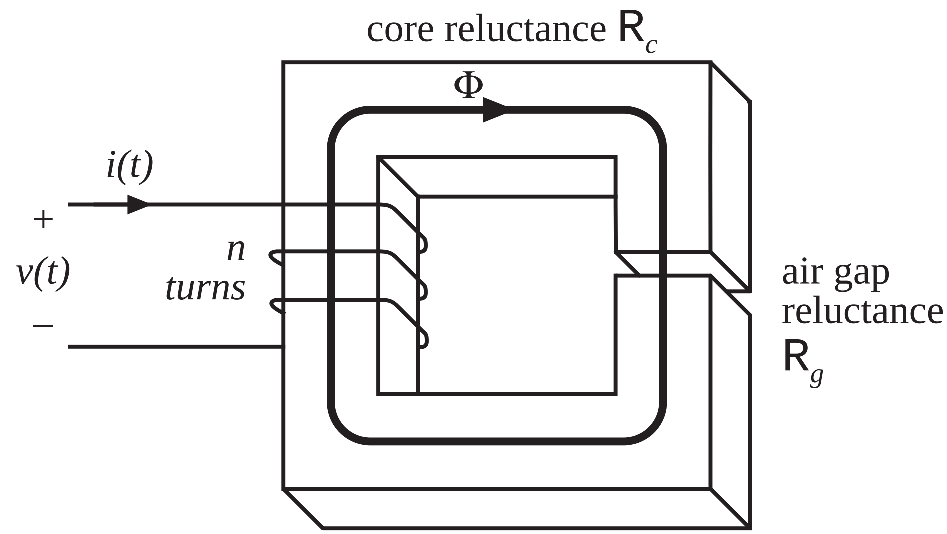


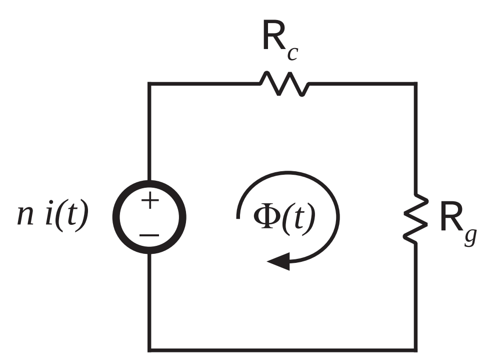




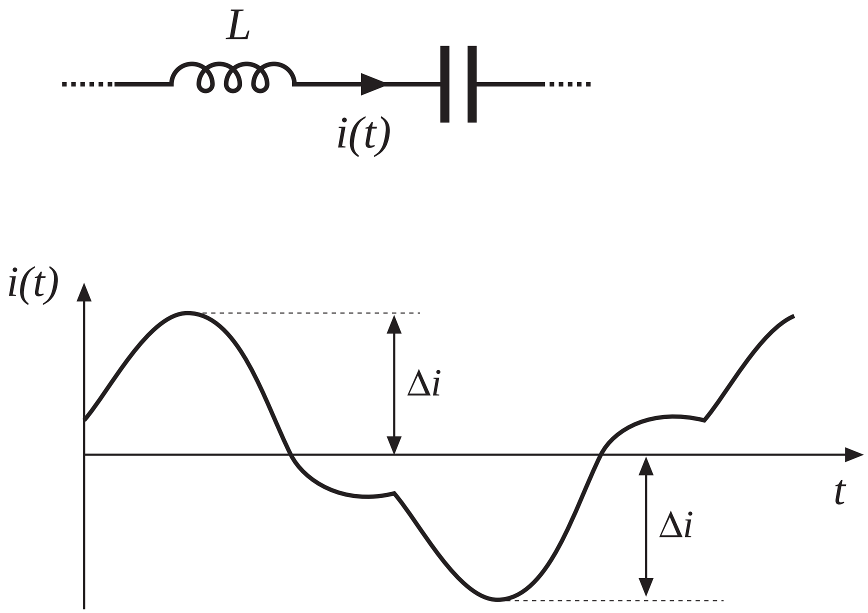
Filter inductor, cont.

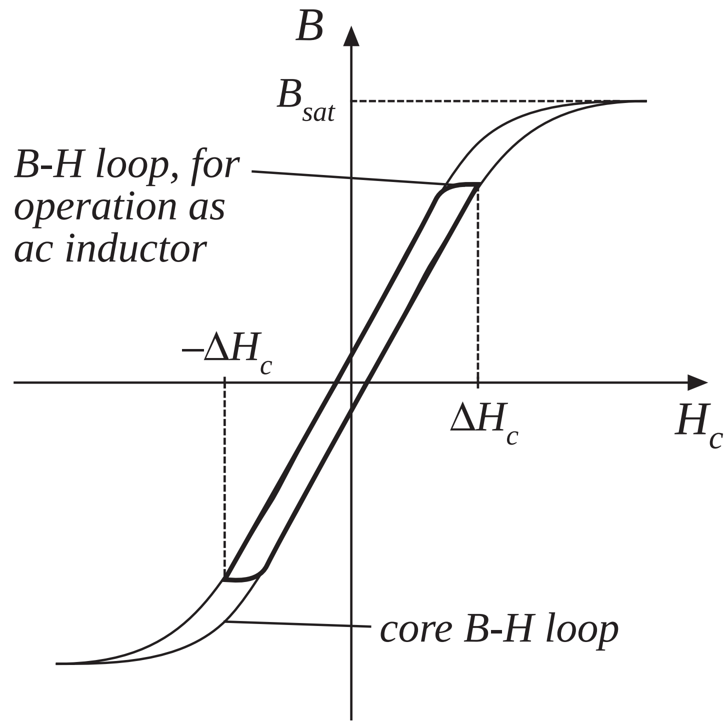
- Negligible core loss, negligible proximity loss
- Loss dominated by dc copper loss
- Flux density chosen simply to avoid saturation
- Air gap is employed
- Could use core materials having high saturation flux density (and relatively high core loss), even though converter switching frequency is high





Ac inductor

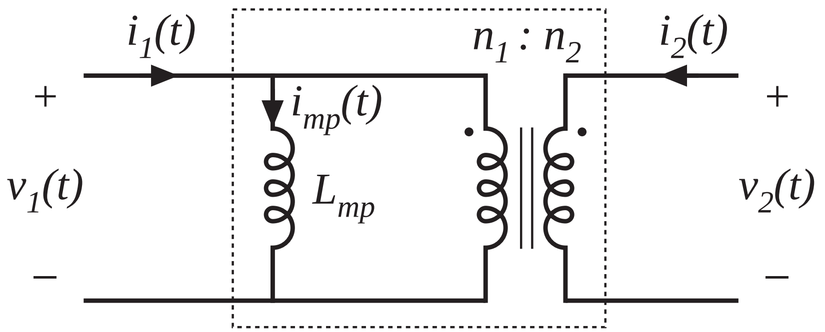

L
i(t)
i(t)
Δi
Δi
t

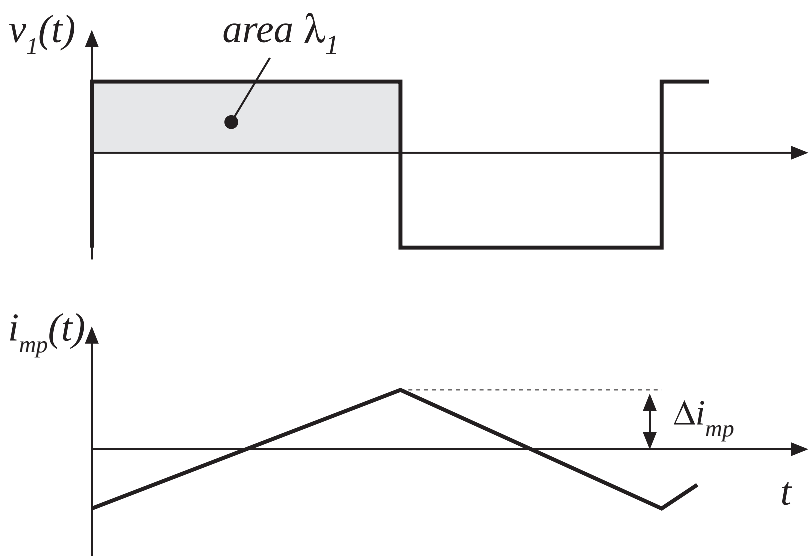

B
B_sat
B-H loop, for operation as ac inductor
–ΔH_c
ΔH_c
H_c
core B-H loop

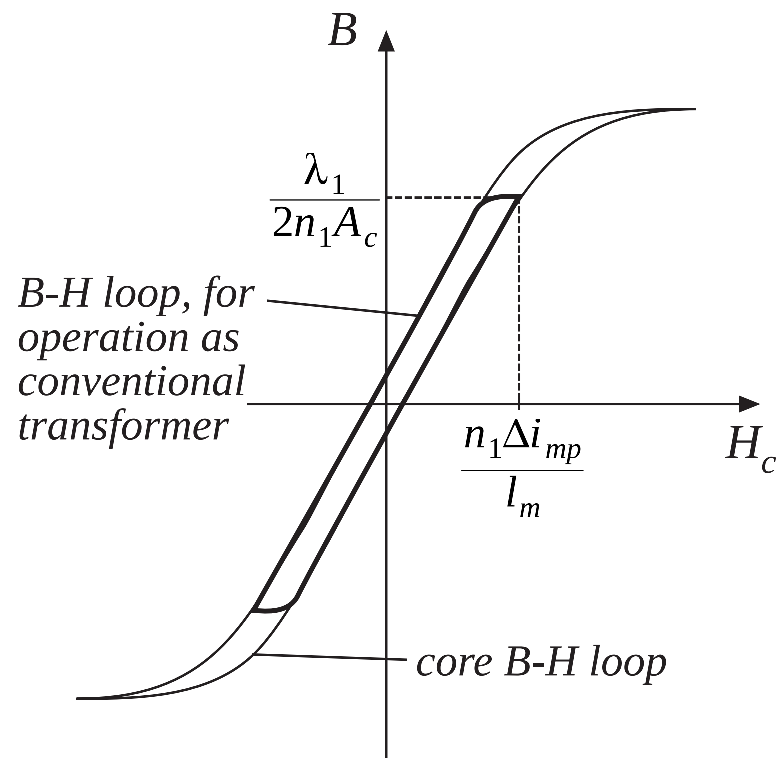
Ac inductor, cont.

- Core loss, copper loss, proximity loss are all significant
- An air gap is employed
- Flux density is chosen to reduce core loss
- A high-frequency material (ferrite) must be employed

Conventional transformer







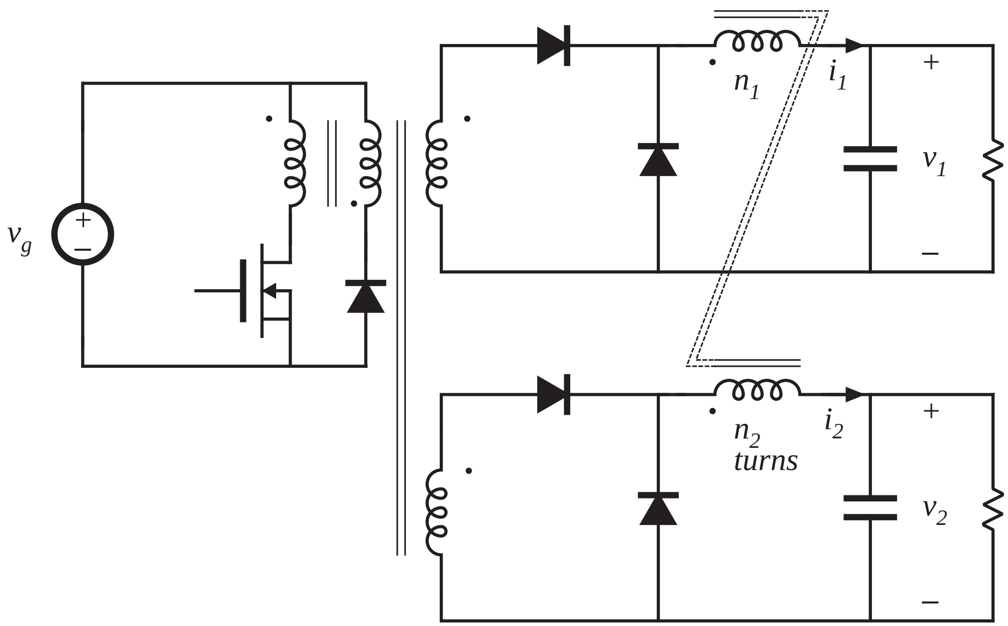
$$H(t) = \frac{n\, i_{mp}(t)}{l_m}$$

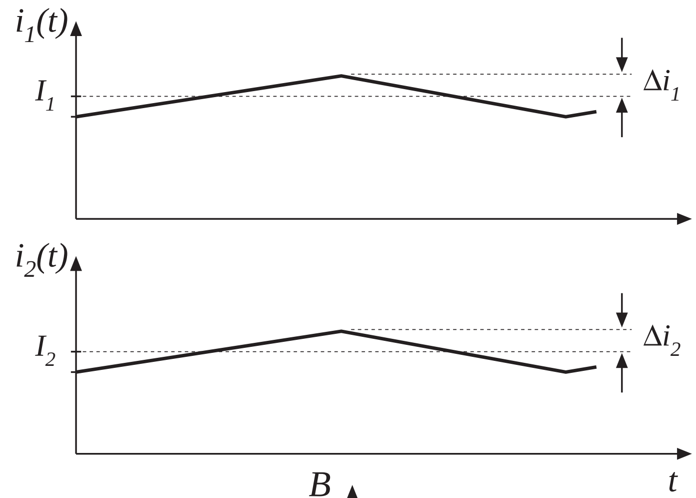
Conventional transformer, cont.

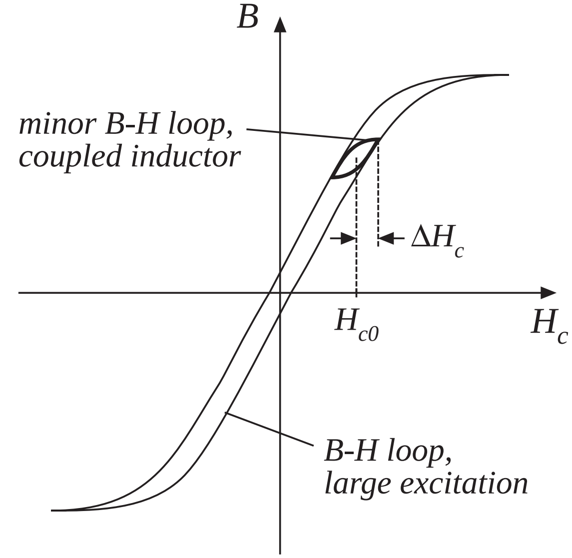
- Core loss, copper loss, and proximity loss are usually significant
- No air gap is employed
- Flux density is chosen to reduce core loss
- A high frequency material (ferrite) must be employed

Coupled inductor

Two-output forward converter example



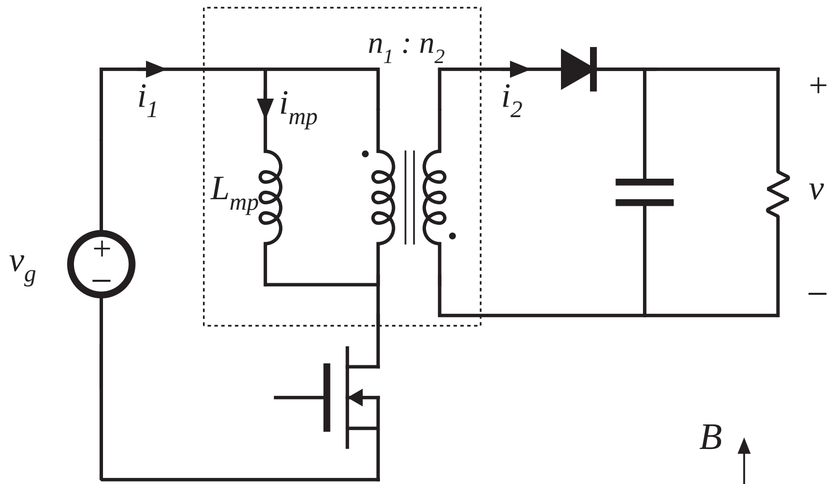


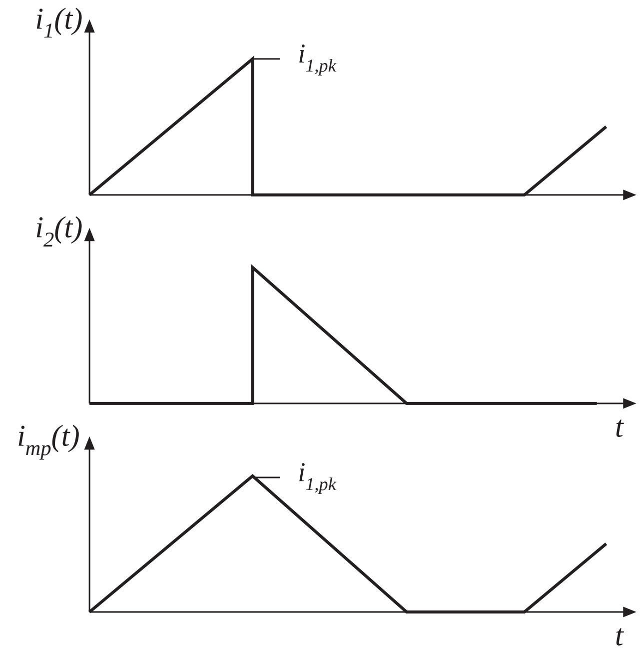


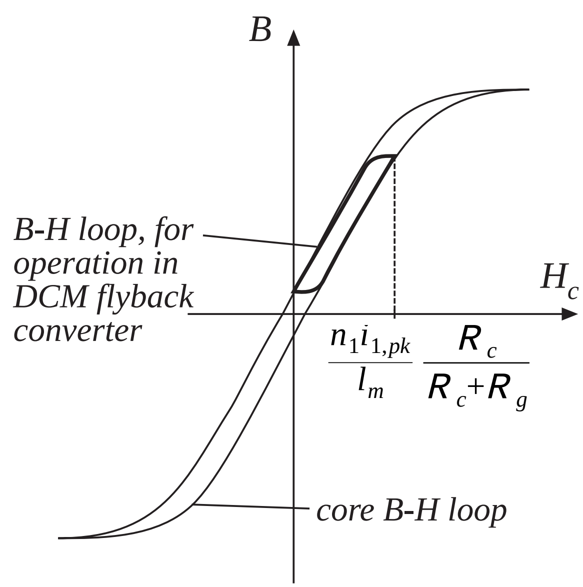
Coupled inductor, cont.

- A filter inductor having multiple windings
- Air gap is employed
- Core loss and proximity loss usually not significant
- Flux density chosen to avoid saturation
- Low-frequency core material can be employed

DCM Flyback transformer



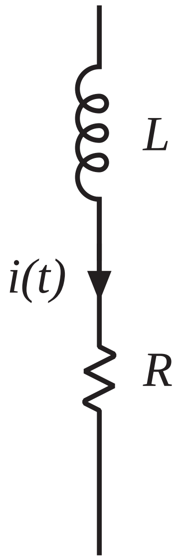




DCM flyback transformer, cont.

- Core loss, copper loss, proximity loss are significant
- Flux density is chosen to reduce core loss
- Air gap is employed
- A high-frequency core material (ferrite) must be used

13.2. Filter inductor design constraints



Objective:

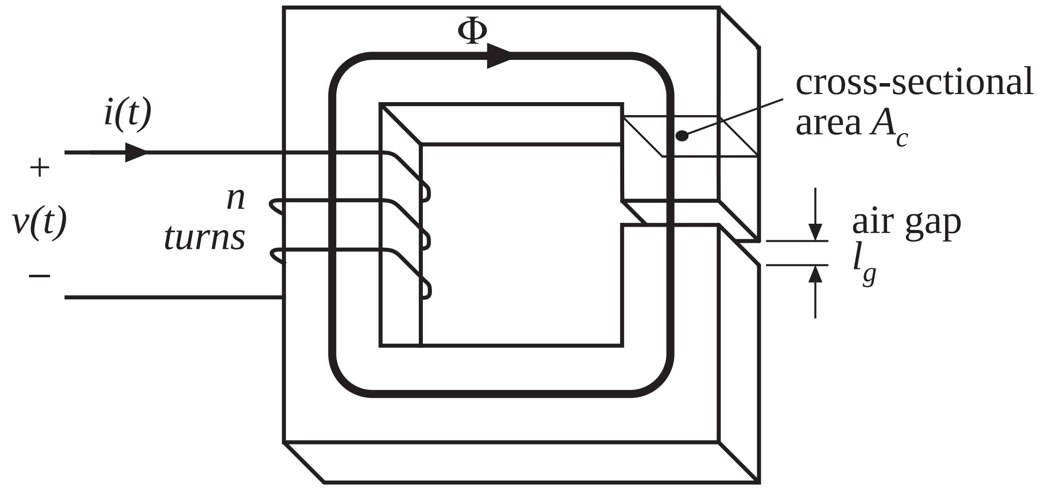
Design inductor having a given inductance L,

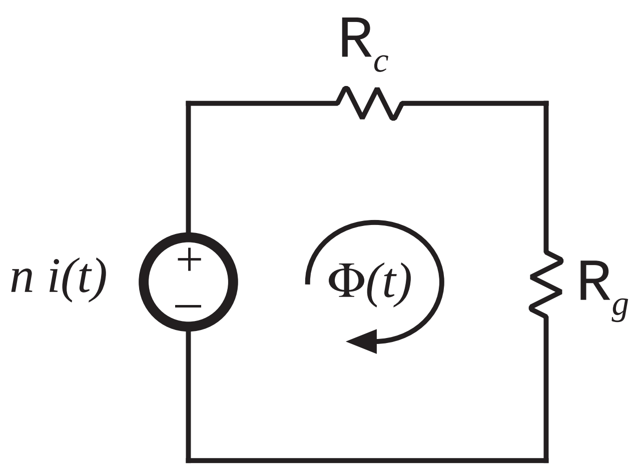
which carries worst-case current I_{max} without saturating,

and which has a given winding resistance R, or, equivalently, exhibits a worst-case copper loss of

$$P_{cu} = I_{rms}^2 R$$

Assumed filter inductor geometry





$$\mathcal{R}_c = \frac{l_c}{\mu_c\, A_c}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0\, A_c}$$

Solve magnetic circuit:

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

For $\mathcal{R}_c >> \mathcal{R}_g$: $\quad ni \approx \Phi\, \mathcal{R}_g$

13.2.1. Constraint: maximum flux density

Given a peak winding current I_{max}, it is desired to operate the core flux density at a peak value B_{max}. The value of B_{max} is chosen to be less than the worst-case saturation flux density of the core material.

From solution of magnetic circuit:

$$ni = BA_c \mathcal{R}_g$$

Let $I = I_{max}$ and $B = B_{max}$:

$$nI_{max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{l_g}{\mu_0}$$

This is constraint #1. The turns ratio n and air gap length l_g are unknown.

13.3.2. Constraint: Inductance

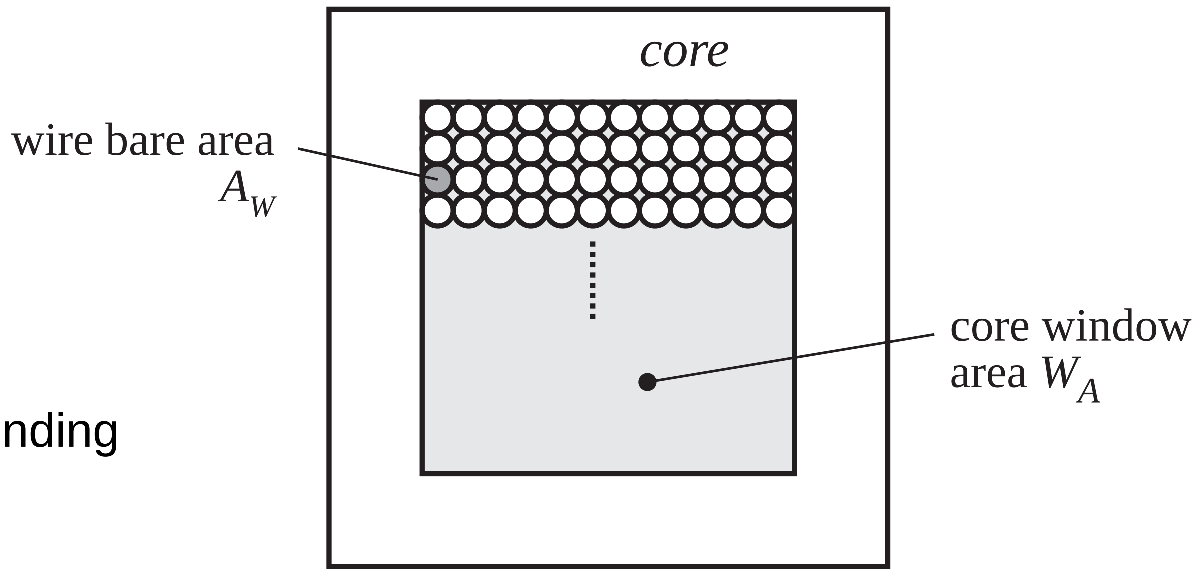
Must obtain specified inductance L. We know that the inductance is

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{l_g}$$

This is constraint #2. The turns ratio n, core area A_c, and air gap length l_g are unknown.

13.3.3. Constraint: Winding area

Wire must fit through core window (i.e., hole in center of core)



Total area of
copper in window:

$$nA_W$$

Area available for winding
conductors:

$$K_u W_A$$

Third design constraint:

$$K_u W_A \geq nA_W$$

The window utilization factor K_u

also called the "fill factor"

K_u is the fraction of the core window area that is filled by copper

Mechanisms that cause K_u to be less than 1:

- Round wire does not pack perfectly, which reduces K_u by a factor of 0.7 to 0.55 depending on winding technique
- Insulation reduces K_u by a factor of 0.95 to 0.65, depending on wire size and type of insulation
- Bobbin uses some window area
- Additional insulation may be required between windings

Typical values of K_u :

0.5 for simple low-voltage inductor

0.25 to 0.3 for off-line transformer

0.05 to 0.2 for high-voltage transformer (multiple kV)

0.65 for low-voltage foil-winding inductor

13.2.4 Winding resistance

The resistance of the winding is

$$R = \rho \frac{l_b}{A_W}$$

where ρ is the resistivity of the conductor material, l_b is the length of the wire, and A_W is the wire bare area. The resistivity of copper at room temperature is $1.724 \cdot 10^{-6}\ \Omega$-cm. The length of the wire comprising an n-turn winding can be expressed as

$$l_b = n\,(MLT)$$

where (MLT) is the mean-length-per-turn of the winding. The mean-length-per-turn is a function of the core geometry. The above equations can be combined to obtain the fourth constraint:

$$R = \rho \frac{n\,(MLT)}{A_W}$$

13.3 The core geometrical constant K_g

The four constraints:

$$nI_{max} = B_{max} \frac{l_g}{\mu_0}$$

$$L = \frac{\mu_0 A_c n^2}{l_g}$$

$$K_u W_A \geq n A_W$$

$$R = \rho \frac{n\,(MLT)}{A_W}$$

These equations involve the quantities

A_c, W_A, and MLT, which are functions of the core geometry,

I_{max}, B_{max} , μ_0, L, K_u, R, and ρ, which are given specifications or other known quantities, and

n, l_g, and A_W, which are unknowns.

Eliminate the three unknowns, leading to a single equation involving the remaining quantities.

Core geometrical constant K_g

Elimination of n, l_g, and A_W leads to

$$\frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

- Right-hand side: specifications or other known quantities
- Left-hand side: function of only core geometry

So we must choose a core whose geometry satisfies the above equation.

The core geometrical constant K_g is defined as

$$K_g = \frac{A_c^2 W_A}{(MLT)}$$

Discussion

$$K_g = \frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

K_g is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where the following quantities are specified:

- Copper loss
- Maximum flux density

How specifications affect the core size:

A smaller core can be used by increasing

$B_{max} \Rightarrow$ use core material having higher B_{sat}

$R \Rightarrow$ allow more copper loss

How the core geometry affects electrical capabilities:

A larger K_g can be obtained by increase of

$A_c \Rightarrow$ more iron core material, or

$W_A \Rightarrow$ larger window and more copper

13.4 A step-by-step procedure

The following quantities are specified, using the units noted:

Wire resistivity	ρ	(Ω-cm)
Peak winding current	I_{max}	(A)
Inductance	L	(H)
Winding resistance	R	(Ω)
Winding fill factor	K_u	
Core maximum flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

Determine core size

$$K_g \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} 10^8 \qquad (\mathrm{cm}^5)$$

Choose a core which is large enough to satisfy this inequality (*see Appendix 2 for magnetics design tables*).

Note the values of A_c, W_A, and MLT for this core.

Determine air gap length

$$l_g = \frac{\mu_0 L I_{max}^2}{B_{max}^2 A_c} 10^4 \qquad \text{(m)}$$

with A_c expressed in cm^2. $\mu_0 = 4\pi 10^{-7}$ H/m.

The air gap length is given in meters.

The value expressed above is approximate, and neglects fringing flux and other nonidealities.

A_L

Core manufacturers sell gapped cores. Rather than specifying the air gap length, the equivalent quantity A_L is used.

A_L is equal to the inductance, in mH, obtained with a winding of 1000 turns.

When A_L is specified, it is the core manufacturer's responsibility to obtain the correct gap length.

The required A_L is given by:

$$A_L = \frac{10 B_{max}^2 A_c^2}{L I_{max}^2} \quad \text{(mH/1000 turns)}$$

Units:

A_c cm²,
L Henries,
B_{max} Tesla.

$$L = A_L\, n^2\, 10^{-9} \quad \text{(Henries)}$$

Determine number of turns n

$$n = \frac{L I_{max}}{B_{max} A_c} \; 10^4$$

Evaluate wire size

$$A_W \le \frac{K_u W_A}{n} \quad (\mathrm{cm}^2)$$

Select wire with bare copper area A_W less than or equal to this value. An American Wire Gauge table is included in Appendix 2.

As a check, the winding resistance can be computed:

$$R = \frac{\rho n\,(MLT)}{A_w} \quad (\Omega)$$

13.5 Summary of key points

1. A variety of magnetic devices are commonly used in switching converters. These devices differ in their core flux density variations, as well as in the magnitudes of the ac winding currents. When the flux density variations are small, core loss can be neglected. Alternatively, a low-frequency material can be used, having higher saturation flux density.

2. The core geometrical constant K_g is a measure of the magnetic size of a core, for applications in which copper loss is dominant. In the K_g design method, flux density and total copper loss are specified.