Load power sources for peak efficiency

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When attempting to load a power source, you might instinctively make the load resistance equal to the Thevenin-equivalent power-supply resistance. This approach, however, yields only 50% efficiency. There's a better way.

A derivation of the maximum efficiency for the simplified circuit shown in Fig 1 leads to the trivial solution of infinite load resistance. A more realistic power source (Fig 2), however, constantly dissipates power through $R_p$.

Now you can find a unique, finite value for $R_L$. Consider the circuit shown in Fig 3. With all resistances represented in terms of the series resistance $R_s$ (i.e., $R_p = \alpha R_s$ and $R_L = \beta R_s$),

\[
\text{OUTPUT POWER} = P_{\text{OUT}} = \left( \frac{V}{R_s} \right)^2 \frac{\alpha^2 \beta R_s}{\alpha + \beta + \frac{\alpha \beta}{\alpha}}
\]

\[
\text{INPUT POWER} = P_{\text{IN}} = \frac{V^2}{R_s} \frac{\alpha + \beta + \frac{\alpha \beta}{\alpha}}
\]

\[
\text{EFFICIENCY} = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\%
\]

\[
= \frac{1}{\beta} + \frac{2}{\alpha} + 1 + \frac{\beta}{\alpha} + \frac{\beta}{\alpha}
\]

\[
f(\beta) = \frac{1}{\beta} + \frac{2}{\alpha} + 1 + \frac{\beta}{\alpha^2} + \frac{\beta}{\alpha}
\]

\[
\frac{df(\beta)}{d\beta} = -\frac{1}{\beta^2} + \frac{2}{\alpha^2} + \frac{1}{\alpha} = 0.
\]

Fig 1—A simplified power-source model produces a trivial solution for load resistance.

Fig 2—A more realistic power-source model constantly dissipates power through $R_p$. 
Thus, the efficiency reaches a maximum value for the value of $\beta$ where $f(\beta)$ has a minimum. This relationship implies that for maximum efficiency,

$$b = \frac{\alpha}{\sqrt{1 + \alpha}}.$$  \hfill (6)

As an example, calculate the value of $R_L$ that provides the maximum efficiency for the circuit shown in Fig 4:

$$R_s = 1 \Omega$$

$$\alpha = \frac{99}{1} = 99$$

$$\therefore \beta = \frac{99}{\sqrt{1 + 99}} = 9.9$$

**MAX EFFICIENCY**

$$= \frac{100\%}{\frac{1}{9.9} + \frac{2}{9.9} + 1 + \frac{9.9}{(99)^2} + \frac{9.9}{99}}$$

$$= 82\%$$

$$R_L = \beta R_s = (9.9)(1) = 9.9\Omega.$$  

Note that when determining $R_s$, you are seeking the open-loop resistance. In a closed loop, the power-source series resistance can appear to be several orders of magnitude less than the actual resistance. As illustrated in Fig 5, the larger you make $\alpha$ (the ratio of parallel resistance to series resistance) the more slowly the efficiency varies with respect to $\beta$. Reducing $\alpha$—increasing parallel losses—lowers the peak efficiency.

Consider the case where $\alpha = 24$. The maximum efficiency obtained would be 67% when $R_L = 4.8\Omega$. If you reduce $R_L$ from 4.8$\Omega$ to 2$\Omega$, the efficiency drops to 59%. However, when you increase $\alpha$ to 10$k$, the maximum efficiency obtained is 98% when $R_L = 100\Omega$. $R_L$ can now vary by $\pm 75\%$ and change the efficiency by no more than $-2.1\%$.